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# M Theory Fivebrane Interpretation for Strong Coupling Dynamics of $SO(N_c)$ Gauge Theories

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## ABSTRACT

We study M theory fivebranes to understand the moduli space of vacua of  $N = 1$  supersymmetric  $SO(N_c)$  gauge theory with  $N_f$  flavors in four dimensions. We discuss how the various branches of this theory arise in the string/M theory brane configurations and compare our results with the ones obtained earlier by Intriligator and Seiberg in the context of field theory. In the M theory approach, we explain the various branches from the asymptotic position of semi-infinite D4 branes in the  $w = x^8 + ix^9$  direction, which is closely related to the eigenvalues of the meson matrix  $M^{ij} = Q_a^i Q_a^j$  where  $Q_a^i$  is a squark multiplet ( $i = 1, \dots, 2N_f$  and  $a = 1, \dots, N_c$ ). In M theory, these branches are explained by observing a new phenomena which did not occur for the gauge groups  $SU(N_c)$  or  $Sp(N_c)$ .

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# 1 Introduction

In the last year much progress has been made in deriving field theory results from string/M theory. Starting with the work of Hanany and Witten for the case of 8 supercharges in three dimensions [1](see also [2]) and its generalization to test the  $N = 1$  field theory dualities in four dimensions [3] in the paper of Elitzur, Gaiotto and Kutasov [4], many important results were obtained, and it turned out to reveal new aspects of both field theory and string theory.

For the  $N = 1$  theories two approaches were developed, one based on the work of [4, 5] and the other based on wrapping D6 branes around three cycles in Calabi-Yau manifolds which was initiated in [6] and generalized in [7]. In all these cases, the theory is in the type IIA set-up with weak string and gauge couplings, and the branes are rigid.

A natural way to go to strong coupling in string theory is to promote the M theory set-up. The M theory approach was pioneered in the seminal work of Witten [8]. A simple observation that both D4 branes and NS5 branes of type IIA setup can be interpreted as a single fivebrane of M theory, gives a unified description. The world volume of the fivebrane can be described as a product space of four dimensional spacetime and the Seiberg-Witten curve [9] for the  $SU$  gauge group. The result was generalized to  $SO$  and  $Sp$  groups [10, 11], and many other very interesting results have been obtained [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27].

In the present work, we continue to follow the lines initiated in [14, 24] and generalized in [22, 26] which exploits the fivebrane of M theory in order to describe strong coupling dynamics of  $N = 1$  theories. We generalize to the case of the  $SO(N_c)$  gauge group. As is known from field theory, for the  $SO$  group there is a clear distinction between the Higgs phase and the confining phase, and furthermore, in different phases, the effective superpotential takes different forms. It is well known [3, 28] that, for  $N_f \leq N_c - 5$ , in quantum theory there exists a superpotential and the theory does not have a ground state. For  $N_f = N_c - 4$  and  $N_c - 3$ , the low energy theory has two branches, one with a superpotential and no ground state, the other without superpotential where there exist ground states. For  $N_f = N_c - 2$ , the unbroken group is  $SO(2)$  and the theory is abelian, and moreover there are vacua with monopole and dyon condensations. For  $N_f \geq N_c - 1$ , there exists a non-abelian magnetic dual theory with the dual gauge group  $SO(N_f - N_c - 4)$ . We find an agreement with field theory results except in some subtle points in which we could not discuss the massless matter which appear at the vacua and in the dual theory (Note that similar problems were encountered in [29] where the  $N = 1$  dualities were derived by breaking  $N = 2$  supersymmetry to  $N = 1$ ).

In section 2, we study the moduli space of  $N = 1$  theory obtained from the  $N = 2$

theory by adding a mass term of the adjoint chiral multiplet. In section 3, we discuss the Higgs branches of the  $N = 2$  theory and the resolution of singularities. In section 4, we rotate the fivebrane to break the supersymmetry to  $N = 1$  for a non-zero value of the adjoint mass and we take the mass of the adjoint to infinity and compare the deformation space of the brane configuration with the moduli space of vacua of the  $N = 1$  theory. Finally in section 5, we will conclude and point out the relevant future directions.

## 2 Field theory results

We give a brief summary of the field theory results by following the same arguments done in [24, 26].

### 2.1 From $N = 2$ theory to $N = 1$ theory

Along the line of [29, 30], we start from  $N = 2$  supersymmetric  $SO(N_c)$  theory with  $2N_f$  squark multiplets  $Q_a^i$  where  $i = 1, \dots, 2N_f$  and the color index  $a = 1, \dots, N_c$ . The superpotential for the massless quarks is

$$W = \sqrt{2} Q_a^i \Phi_{ab} Q_b^j J_{ij} \quad (2.1)$$

where  $\Phi_{ab}$  is a chiral multiplet in the adjoint representation of the gauge group, and  $J$  is the symplectic  $2N_f \times 2N_f$  dimensional metric used to raise or lower flavor indices. We want to break  $N = 2$  supersymmetry to  $N = 1$  by inserting a mass term of the adjoint field  $\Phi_{ab}$ . The superpotential (2.1) then modifies to

$$W = \sqrt{2} Q_a^i \Phi_{ab} Q_b^j + \mu \text{Tr}(\Phi^2). \quad (2.2)$$

We can integrate out the  $\Phi_{ab}$  by using its F-term equation:

$$Q_a^i J_{ij} Q_b^j - \sqrt{2} \mu \Phi_{ab} = 0. \quad (2.3)$$

As the mass  $\mu$  is increased beyond the scale of the asymptotic free  $N = 2$  theory, and we integrate out the adjoint field, by one loop matching condition between the  $N = 1$  and  $N = 2$  theory, we get the dynamical scale  $\Lambda_{N=1}$  given by:

$$\Lambda_{N=1}^{3(N_c-2)-2N_f} = \mu^{N_c-2} \Lambda_{N=2}^{2(N_c-2)-2N_f}. \quad (2.4)$$

Plugging (2.3) into (2.2) we can write down the expression of the superpotential only in terms of  $M^{ij} = Q_a^i Q_a^j$ . The final form of the superpotential is given by:

$$\Delta W = \frac{1}{2\mu} \text{Tr}(M J M J). \quad (2.5)$$

The system below the energy scale  $\mu$  can be considered as  $N = 1$  theory with the tree level superpotential (2.5) and with the dynamical scale  $\Lambda_{N=1}$  given by (2.4). In the next two subsections we will discuss the gauge group  $SO(2N_c)$  and  $SO(2N_c + 1)$  for various values of  $N_f$  and  $N_c$ .

## 2.2 $SO(2N_c)$ groups

Our discussion is based on the field theory results obtained earlier in [3, 28]. Since the number of flavors is  $2N_f$ , the difference between the number of flavors and the number of colors is always even.

- $0 \leq 2N_f \leq 2N_c - 6$

A supepotential is dynamically generated by strong coupling effects [31]. For a generic value of  $2N_f$ , the ADS superpotential is given by

$$W_{ADS} = (N_c - N_f - 1)\omega_{2N_c-2N_f-2}\left(\frac{16\Lambda_{N=1}^{6N_c-2N_f-6}}{\det M}\right)^{\frac{1}{2N_c-2N_f-2}} \quad (2.6)$$

where  $\omega_{2N_c-2N_f-2}$  denotes  $(2N_c - 2N_f - 2)$ -th root of unity. For large but finite  $\mu$ , at the scale far below  $\mu$  but larger than  $\Lambda_{N=1}$ , the superpotential (2.5) can be regarded as a perturbation of the ordinary  $N = 1$  theory. Thus the total effective superpotential is

$$W_{eff} = W_{ADS} + \Delta W. \quad (2.7)$$

As explained in [24, 26], this form for  $W_{eff}$  is exact for any non-zero values of  $\mu$  by holomorphy and symmetry argument. Because the meson field  $M$  is a symmetric matrix it can be brought to diagonal form after an appropriate similarity transformation, where the diagonal elements are given by  $(m_1, \dots, m_{N_f}, m_1, \dots, m_{N_f})$ . By extrematizing  $W_{eff}$  with this explicit form of  $M$ , all the diagonal elements of  $M$  become equal and are given by:

$$m = 2^{\frac{2}{2N_c-2-N_f}}\mu\Lambda_{N=2}. \quad (2.8)$$

The value of  $m$  in (2.8) describes the moduli space of the  $N = 1$  theory in the presence of the perturbation to the ADS superpotential. In the limit of  $\mu \rightarrow \infty$  keeping  $\Lambda_{N=1}$  finite,  $m$  diverges so that  $\Delta W$  goes to 0. Thus there is no supersymmetric vacua.

- $2N_f = 2N_c - 4$

As explained already in [28], there exist two branches of the theory. The origin of these branches is due to the fact that  $SO(4)$  is isomorphic to the product  $SU(2)_L \times$

$SU(2)_R$  and the gaugino condensation in the product gauge group generates the superpotential:

$$W_{ADS} = \frac{1}{2}(\epsilon_L + \epsilon_R) \left( \frac{16\Lambda_{N=1}^{2(2N_c-1)}}{\det M} \right)^{1/2}, \quad (2.9)$$

where  $\epsilon_{L,R}$  are  $\pm 1$ . The case of  $\epsilon_L \times \epsilon_R = 1$  gives the first branch while the case of  $\epsilon_L \times \epsilon_R = -1$  does the second one. The first branch is the continuation of the (2.7) to the case  $2N_f = 2N_c - 4$ . By taking the extremum with respect to  $M$ , which has diagonal form, we obtain again  $m = (4(N_c - 2))^{\frac{1}{N_c}} \mu \Lambda_{N=2}$  which will lead to the same limit with no vacuum as previous case when  $\mu$  goes to infinity. However, the second branch is not a continuation of (2.7). For this case,  $W_{ADS} = 0$  and the only thing that is to be extrematized is  $\Delta W$  which just gives  $m = 0$ . So there is a vacuum at the origin, i.e.  $M = 0$ . This is in complete agreement with the theory [28] without the perturbation of  $\Delta W$ . So quantum mechanically, at the origin only the  $M$  quanta are massless and this describes the confinement of the elementary degrees of freedom. The global chiral symmetry is unbroken, so we have confinement without chiral symmetry breaking, a new feature which did not appear in the case of  $SU$  gauge group with  $N_f = N_c$ .

- $2N_f = 2N_c - 2$

There is no superpotential because in (2.6) the inverse power of the right hand side vanishes,  $2N_c - 2N_f - 2 = 0$  and the theory has a quantum moduli space of vacua given by the expectation values of  $M^{ij}$ . The gauge group  $SO(2N_c)$  is broken to  $SO(2)$  and the theory is in a Coulomb phase with a massless supermultiplet. As a continuation of the previous case, there exist two branches, one at  $m \sim \mu \Lambda_{N=2}$ , the other at  $m = 0$ . Using now the RG matching relation (2.4),  $\Lambda_{N=1}^{4N_c-4} = \mu^{2N_c-2} \Lambda_{N=2}^{2N_c-2}$ , we get  $\det M \sim \Lambda_{N=1}^{2N_c-4}$  which is exactly the same as  $U_1$  obtained in [28]. The other vacua is at  $\det M = 0$  which is again in accordance with  $U = 0$ . At this stage, let us stress that we could not obtain any information about the monopoles or dyon condensation, but we could check the existence of the vacua of the theory.

- $2N_f \geq 2N_c$

In this case, very interesting phenomena have been observed in [28, 3]. The infrared behavior of the theory has a dual, magnetic description in terms of an  $SO(2N_f - 2N_c + 4)$  gauge theory with  $2N_f$  flavors of dual quarks and an additional gauge singlet field  $M^{ij}$ . A superpotential takes the form of

$$W = \frac{1}{2\lambda} M^{ij} q_i q_j \quad (2.10)$$

where  $\lambda$  relates the scale of electric theory  $\Lambda_{N=1}$  and those of the magnetic theory  $\tilde{\Lambda}_{N=1}$

by

$$\Lambda^{3(N_c-1)-N_f} \tilde{\Lambda}^{3(N_f-N_c+1)-N_f} = C(-1)^{N_f-N_c} \mu^{N_f} \quad (2.11)$$

Besides, there are also gauge invariant operators given in the electric theory by

$$\begin{aligned} B^{[i_1, \dots, i_{2N_c}]} &= Q^{i_1} \dots Q^{i_{2N_c}} \\ b^{[i_1, \dots, i_{2N_c-4}]} &= W_\alpha^2 Q^{i_1} \dots Q^{i_{2N_c-4}} \\ \mathcal{W}_\alpha^{[i_1, \dots, i_{2N_c-2}]} &= W_\alpha Q^{i_1} \dots Q^{i_{2N_c-2}} \end{aligned} \quad (2.12)$$

Actually, the last two types of baryons,  $b$  and  $\mathcal{W}$ , occur in the case of mixed Coulomb-Higgs branches. This may appear when some vev for  $Q$ 's are not zero but not all of them are different from zero. Our values for the vev of  $M$  tells us that all of them can be zero (for Coulomb phase) or be non-zero (for Higgs phase). So there is no mixed Coulomb-Higgs branches. That is, in our theory  $b$  and  $\mathcal{W}$  are 0 identically, so only baryons of type  $B$  do exist.

### 2.3 $SO(2N_c + 1)$ case

For  $2N_f \leq (2N_c + 1) - 5$ , we can continue to proceed with the arguments as in the case of  $2N_f \leq 2N_c - 6$  and obtain the moduli space given by the vev of  $M$ . For  $2N_f = (2N_c + 1) - 3$ , again there exist two branches, as explained in [28]. The ADS superpotential is

$$W = 4(1 + \epsilon) \frac{\Lambda_{N=1}^{4N_c-1}}{\det M} \quad (2.13)$$

so we have again two physically inequivalent phase branches as a function of  $\epsilon$ . For  $\epsilon = 1$ , this is just a continuation of (2.7) to the case  $2N_f = (2N_c + 1) - 3$ , so there is no vacuum as  $\mu$  goes to infinity. For  $\epsilon = -1$  there is no ADS superpotential. Again, what is to be extrematized is  $\Delta W$  which gives as moduli space solution only  $m = 0$ . In order to match the 't Hooft anomaly conditions, we need to have supplementary fields  $q_i$  at the origin,  $M = 0$ , which have mass away from  $M = 0$ . The most general invariant superpotential which describes the coupling of  $q$ 's and  $M$  can be written as [28]

$$W = \frac{1}{2\lambda} f\left(t = \frac{(\det M)(M^{ij} q_i q_j)}{\Lambda_{N=1}^{2N_c-2}}\right) M^{ij} q_i q_j \quad (2.14)$$

where  $f(t)$  must be holomorphic near  $t = 0$  and  $f(0) = 1$ . The massless fields  $q_i$  are identified with the “exotic” composites  $b_i$  as in [28], and they confine at  $M = 0$  giving again confinement without chiral symmetry breaking. For  $2N_f \geq 2N_c - 4$  there is a dual description of the theory with gauge group  $SO(2N_f - 2N_c - 4)$ , with dual quarks  $q^i$  in the fundamental of the magnetic group and with singlets  $M^{ij}$ .

We close this section by noting that for  $SO$  group we do not have non-baryonic branches as compared with  $SU$  group because of the specific form of the solution for  $Q$ 's which solve the D-term and F-term equations [29]. This simplifies our discussion as compared with  $SU$  group.

### 3 $N = 2$ Higgs Branch from M Theory

We study the moduli space of vacua of  $N = 2$  supersymmetric  $SO(N_c)$  gauge theory with  $N_f$  flavors by analyzing M theory fivebrane. Let us first describe the Higgs branch in the type IIA brane configuration and later go on to the M theory fivebrane in terms of geometrical picture.

The brane configuration consists of two parallel NS5 branes extending in the direction  $(x^0, x^1, x^2, x^3, x^4, x^5)$ , the D4 branes stretched between two NS5 branes and extending over  $(x^0, x^1, x^2, x^3)$  and being finite in  $x^6$  direction, and the D6 branes extending in the direction of  $(x^0, x^1, x^2, x^3, x^7, x^8, x^9)$ . In order to get orthogonal gauge group, we should consider an O4 orientifold which is parallel to the D4 branes in order not to break the supersymmetry and is infinite extent along the  $x^6$  direction. The O4 orientifold gives a spacetime reflection as  $(x^4, x^5, x^7, x^8, x^9) \rightarrow (-x^4, -x^5, -x^7, -x^8, -x^9)$ , in addition to the gauging of worldsheet parity  $\Omega$ . Each object which does not lie at the fixed points of the spacetime symmetry ( i.e., over the O4 orientifold ) must have its mirror image. Thus NS5 branes have a mirror in  $(x^4, x^5)$  directions and D6 branes have a mirror in  $(x^7, x^8, x^9)$  directions. For  $SO(2N_c)$  gauge group, each D4 brane at  $v = x^4 + ix^5$  has its mirror image at  $-v$ :  $N_c$  D4 branes and its mirror  $N_c$  ones. Similarly, for the  $SO(2N_c+1)$  gauge group, there exist an extra single D4 brane which lies over the O4 orientifold and is frozen at  $v = 0$  because it does not contain its mirror image, as well as  $N_c$  D4 branes and its mirror  $N_c$  ones. Another important ingredient of O4 orientifold is its charge, which is related to the sign of  $\Omega^2$ . When the D4 brane carries one unit of this charge, the charge of the O4 orientifold is  $\pm 1$ , for  $\Omega^2 = \mp 1$  in the D4 brane sector.

To go to the Higgs branch, the D4 branes are broken on the D6 branes and suspended between those D6 branes being allowed to move on the directions  $(x^7, x^8, x^9)$ . The dimension of the Higgs moduli space is obtained by counting all possible breakings of D4 branes into D6 branes as follows: the first D4 brane is broken in  $N_f - 1$  sectors between the D6 branes, the second D4 branes is broken in  $N_f - 3$  sectors and so on. Besides that we have to consider the symmetric orientifold projection which increases degrees of freedom, as explained in [32]. Then the dimension of the Higgs moduli space is given by for  $SO(N_c)$ :

$$2[(2N_f - 2 + 1) + (2N_f - 6 + 1) + \cdots + (2N_f - 2N_c + 2 + 1)] = N_c(2N_f - N_c + 1) \quad (3.1)$$

or  $2N_c N_f - N_c(N_c - 1)$  where we have explicitly added 1 as a result of the symmetric orientifold projection. The overall factor 2 in the left hand side is due to the mirror D6 branes and the result is very similar to the field theory result where because of the  $N_f$  vevs, the gauge symmetry is completely broken and there are  $N_c N_f - \frac{1}{2} N_c(N_c - 1)$  massless neutral hypermultiplets. Thus, the field theory results match the brane configuration results. There is no intersection between the Higgs and Coulomb branches and we have only *one Higgs branch*.

Let us describe how the above brane configuration is embedded in M theory in terms of a single M fivebrane whose worldvolume is  $\mathbf{R}^{1,3} \times \Sigma$ , where  $\Sigma$  is identified with Seiberg-Witten curves that determine the solutions to Coulomb branch of the field theory. As usual, we write  $s = (x^6 + ix^{10})/R$ ,  $t = e^{-s}$ , where  $x^{10}$  is the eleventh coordinate and is compactified on a circle of radius  $R$ . Then the curve  $\Sigma$ , describing  $N = 2$   $SO(N_c)$  gauge theory with  $N_f$  flavors and even  $N_c$ , is given[10] by an equation in  $(v, t)$  space

$$v^2 t^2 - B(v^2, u_k) t + \Lambda_{N=2}^{2N_c-4-2N_f} v^2 \prod_{i=1}^{N_f} (v^2 - m_i^2) = 0. \quad (3.2)$$

Here  $B(v^2, u_k)$  is a degree  $N_c$  polynomial of the form  $v^{N_c} + u_2 v^{N_c-2} + \dots + u_{N_c}$  with only even degree terms and the coefficients depending on the moduli  $u_k$ , and  $m_i$  is the mass of quark. Similarly, for odd  $N_c$ , it takes the form of

$$v t^2 - B(v^2, u_k) t + \Lambda_{N=2}^{2N_c-4-2N_f} v \prod_{i=1}^{N_f} (v^2 - m_i^2) = 0 \quad (3.3)$$

where this time  $B(v^2)$  is a polynomial of degree  $N_c - 1$ .

### 3.1 Including D6 Branes

In M theory, the type IIA D6 branes are the magnetic dual of the electrically charged D0 branes, which are the Kaluza-Klein monopoles described by a Taub-NUT space. We will ignore the hyper-Kähler structure of this Taub-NUT space and instead use one of the complex structures, which can be described by [10]

$$yz = \Lambda_{N=2}^{2N_c-4-2N_f} \prod_{i=1}^{N_f} (v^2 - m_i^2) \quad (3.4)$$

in  $\mathbf{C}^3$  for  $SO(N_c)$ . The D6 branes are located at  $y = z = 0, v = \pm m_i$ . This surface, which represents a nontrivial  $\mathbf{S}^1$  bundle over  $\mathbf{R}^3$  instead of the flat four dimensional space  $\mathbf{R}^3 \times \mathbf{S}^1$  with coordinates  $(x^4, x^5, x^6, x^{10})$ , is the unfolding of the  $A_{2n-1}$  ( $n = N_f$ )



singularity in general. The Riemann surface  $\Sigma$  is embedded as a curve in this curved surface and given by

$$v^2(y + z) = B(v^2) \quad (3.5)$$

when  $N_c$  is even. In the case  $N_c$  is odd, the curve is given by

$$v(y + z) = B(v^2). \quad (3.6)$$

Our type IIA brane configuration has  $U(1)_{4,5}$  and  $SU(2)_{7,8,9}$  symmetries regarded as classical  $U(1)$  and  $SU(2)$  R-symmetry groups of the 4 dimensional theory on the brane worldvolume. One of them,  $SU(2)_{7,8,9}$ , is preserved only in M theory quantum mechanical configuration, but  $U(1)_{4,5}$  is broken. The  $U(1)_R$  symmetry of the  $N = 2$  supersymmetric field theory is anomalous being broken by instantons whose factor is proportional with  $\Lambda_{N=2}^{2N_c-4-2N_f}$ . We can easily see that the charge of  $\Lambda_{N=2}^{2N_c-4-2N_f}$  is  $(4N_c - 8 - 2N_f)$  from equations (3.4) by considering  $v$  of charge 2.

### 3.2 Resolution of Singularities and the Higgs Branch

In this section, we follow the notations of [26] and refer to [26] for details. When all the bare masses are the same but not zero (say  $m = m_i$ ), the surface (3.4) develops  $A_{N_f-1}$  singularities at two points  $y = z = 0, v = \pm m$ . Over each singular point, there are  $(N_f - 1)$  rational curves on the resolved surface. We denote the rational curves over the point  $y = z = 0, v = m$  by  $C_1, C_2, \dots, C_{N_f-1}$  and those over the point  $y = z = 0, v = -m$  by  $C'_1, C'_2, \dots, C'_{N_f-1}$ . When we turn off the bare mass, i.e.,  $m_i = 0$  for all  $i$ , the singularity gets worse and a new rational curve appears in the resolved surface which we call  $E$ .

Now, we would like to study the Higgs branch when all the bare masses are turned off. As noticed in [26, 24], the complex structure of Taub-NUT space develops  $A$ -type singularities when all bare masses become massless. In M theory, the Higgs branch appears when the fivebrane intersects with the D6-branes. As a special case, we will consider when all D4 branes are broken on D6 branes in the type IIA picture. To describe the Higgs branch, we will study how the curve

$$y + z = v^{N_c} \quad (3.7)$$

appears in the resolved  $A_{2N_f-1}$  surface since there are  $N_c$  D4 branes in this configuration. Away from the singular point  $y = z = v = 0$ , we may regard the curve as embedded in the original  $y - z - v$  space because there is no change in the resolved surface in this region. Near the singular point  $y = z = v = 0$ , we have to consider the resolved surface. On the  $i$ -th patch  $U_i$  of the resolved surface, the equation of the curve  $\Sigma$  becomes

$$y_i^i z_i^{i-1} + y_i^{2N_f-i} z_i^{2N_f+1-i} = y_i^{N_c} z_i^{N_c} \quad (3.8)$$

By factorizing this equation according to the range of  $i$ , we will see that the curve consists of several components. One component, which we call  $C$ , is the extension of the one in the region away from  $y = z = v = 0$  which we have already considered. The other components are the rational curves  $C_1, \dots, C_{n-1}, E, C'_1, \dots, C'_{n-1}$  with some multiplicities. For convenience, we rename the exceptional curves  $E_1, \dots, E_{2n-1}$  so that  $E_i$  is defined by  $y_i = 0$  on  $U_i$  and  $z_{i+1} = 0$  on  $U_{i+1}$ . Hence we can see from the above factorization that the component  $E_i$  has multiplicity  $l_i = i$  for  $i = 1, \dots, N_c$ ;  $l_i = N_c$  for  $i = N_c + 1, \dots, 2N_f - N_c$ ; and  $l_i = 2N_f - i$  for  $i = 2N_f - N_c + 1, \dots, 2N_f - 1$ . Note that the component  $C$  intersects with  $E_{N_c}$  and  $E_{2N_f - N_c}$ .

To count the dimension of the Higgs branch, recall that once the curve degenerates and  $\mathbf{CP}^1$  components are generated, they can move in the  $x^7, x^8, x^9$  directions [8]. This motion together with the integration of the chiral two-forms on such  $\mathbf{CP}^1$ 's parameterizes the Higgs branch of the four-dimensional theory. While the components  $C_i$  and  $C'_i$  have to move in pairs due to orientifolding, the middle component  $E_{N_f} = E$  moves freely because it is mirror symmetric with respect to O4 plane. Thus the quaternionic dimension of the Higgs branch is

$$\frac{1}{2} \left( \sum_{i=1}^{2N_f-1} l_i + l_{N_f} \right) = \sum_{i=1}^{N_c-1} i + (2N_f - 2N_c + 2)N_c = \frac{1}{2} N_c (2N_f - N_c + 1), \quad (3.9)$$

which is the half of the complex dimension given in (3.1).

## 4 $N = 1$ SQCD

### 4.1 The Rotated Configuration

By adding a mass term to the adjoint chiral multiplet,  $N = 2$  SUSY will be broken to  $N = 1$ . To describe the corresponding brane configuration in M theory, we introduce a complex coordinate

$$w = x^8 + ix^9. \quad (4.1)$$

Before breaking the  $N = 2$  supersymmetry, the fivebrane is located at  $w = 0$ . Now we rotate only the left NS5 brane toward the  $w$  direction and from the behavior of two asymptotic regions which correspond to the NS5(left) and NS'5(right) branes with  $v \rightarrow \infty$  this rotation leads to the following boundary conditions for  $SO(N_c)$ :

$$\begin{aligned} w &\rightarrow \mu v \quad \text{as } v \rightarrow \infty, \quad t \sim v^{N_c-2} \\ w &\rightarrow 0 \quad \text{as } v \rightarrow \infty, \quad t \sim \Lambda_{N=2}^{2(N_c-2-N_f)} v^{2N_f-N_c+2}. \end{aligned} \quad (4.2)$$

After rotation,  $SU(2)_{7,8,9}$  is broken to  $U(1)_{8,9}$ . In order for this to be consistent, because of the relation between  $v$  and  $w$  in (4.2),  $\mu$  should have charges under  $U(1)_{4,5} \times U(1)_{8,9}$ . That is,  $v$  has charge 2 under  $U(1)_{4,5}$  and 0 under  $U(1)_{8,9}$  while  $w$  has charge 0 under  $U(1)_{4,5}$  and 2 under  $U(1)_{8,9}$ .

Since the rotation is only possible at points in moduli space at which all 1-cycles on the curve  $\Sigma$  are degenerate [9], the curve  $\Sigma$  is rational, which means that the functions  $v$  and  $t$  can be expressed as some rational functions of  $w$  after we identify  $\Sigma$  with a complex plane  $w$  having some deleted points. Because of the symmetry of  $v \rightarrow -v$  and  $w \rightarrow -w$  due to orientifolding, we can write them as:

$$v^2 = P(w^2), \quad t = Q(w). \quad (4.3)$$

(Of course, we may write  $t = Q(w^2)$  for even  $N_c$  and  $t^2 = Q(w^2)$  for odd  $N_c$  considering the extra symmetry  $t \rightarrow -t$ . But we suppressed these extra symmetries to treat both cases uniformly.) Since  $v$  and  $t$  become  $\infty$  only if  $w = 0$  and  $\infty$  from the boundary conditions, these rational functions are some polynomials of  $w$  up to a factor of some power of  $w$ :  $P(w^2) = w^{2a}p(w^2)$  and  $Q(w) = w^bq(w)$  where  $a$  and  $b$  are some integers and  $p(w^2)$  and  $q(w)$  are some polynomials of  $w$  which we may assume non-vanishing at  $w = 0$ . Near one of the points at  $w = \infty$ ,  $v$  and  $t$  behave as  $v \sim \mu^{-1}w$  and  $t \sim v^{N_c-2}$  by (4.2). Thus the rational functions are of the form

$$P(w^2) = w^{2a}(w^{2-2a} + \dots)/\mu^2 \quad \text{and} \quad Q(w) = \mu^{-N_c+2}w^b(w^{N_c-2-b} + \dots) \quad (4.4)$$

for  $SO(N_c)$ . Around a neighborhood  $w = 0$ , the Riemann surface  $\Sigma$  can be parameterized by  $1/v$  which vanishes as  $w \rightarrow 0$  from the boundary condition. Since  $w$  and  $1/v$  are two coordinates around the neighborhood  $w = 0$  in the compactification of  $\Sigma$  and vanish at the same point, they must be linearly proportional to each other  $w \sim 1/v$  in the limit  $w \rightarrow 0$ . The numerator  $w^{2-2a} + \dots$  of  $P(w^2)$  then takes the form  $(w^2 - w_+)(w^2 - w_-)$ . But the equation of  $v^2 = P(w^2)$  implies that  $P(w^2)$  must be a square of a rational function. Hence we have  $w_+ = w_-$  and by letting  $w_0^2 = w_{\pm}$

$$v^2 = P(w^2) = \frac{(w^2 - w_0^2)^2}{\mu^2 w^2}. \quad (4.5)$$

Since  $t \sim v^{2N_f - N_c + 2}$  and  $w \sim 1/v$  as  $w \rightarrow 0$ , we get  $b = N_c - 2 - 2N_f$  and thus,

$$t = Q(w) = \mu^{-N_c+2}w^{N_c-2-2N_f}(w^{2N_f} + \dots). \quad (4.6)$$

By the equation  $yz = v^{2N_f}$  where  $N_f > 0$  defining the space-time,  $t = 0$  (i.e.,  $y = 0$ ) implies  $v = 0$ . Therefore the zeros of the polynomial  $w^{2N_f} + \dots$  are equal to the zeros  $\pm w_0$  of  $P(w^2)$ . Hence we conclude

$$t = Q(w) = \mu^{-N_c+2}w^{N_c-2-2N_f}(w^2 - w_0^2)^{N_f}. \quad (4.7)$$

Near  $w = \pm w_0$ ,  $v$  and  $t$  will satisfy the relation (3.2) (resp. (3.3)) for even (resp. odd)  $N_c$  which is approximated in the limit  $v \rightarrow \infty$  by

$$t^2 - v^{N_c-2}t + \Lambda_{N=2}^{2N_c-4-2N_f} v^{2N_f} = 0 \quad (4.8)$$

By plugging the equations (4.5) and (4.7) in the this equation and considering the limit  $w \rightarrow \pm w_0$ , we obtain the expression for  $w_0$ :

$$(-1)^{N_f} \Lambda_{N=2}^{2N_c-4-2N_f} w_0^{2N_f} \mu^{2N_c-4-2N_f} = w_0^{2N_c-4} \quad (4.9)$$

This yields a first solution

$$w_0 = (-1)^{\frac{N_f}{2N_c-4-2N_f}} \mu \Lambda_{N=2}. \quad (4.10)$$

and a second possible solution

$$w_0 = 0 \quad (4.11)$$

As we have noticed in section 2, if  $2N_f = N_c - 4$  (resp.  $2N_f = N_c - 3$ ) for even (resp. odd)  $N_c$ , the global chiral symmetry is not broken at the origin so there exists a confinement at the origin without chiral symmetry breaking. Now, by sending the D6 branes to infinity in the  $x_6$  direction, there are  $N_f$  semi-infinite D4 branes ending on the right NS5 brane from the right, whose asymptotic positions in the  $w$  direction are just given by the eigenvalues of the meson matrix  $M$ . In the  $Sp$  case, the solution (4.10) was the only one taken in [26]. The solution of (4.11) corresponds to a zero eigenvalue of  $M$  (the origin) and the chiral symmetry is broken at the origin for  $Sp$  case so  $w_0 = 0$  is not an acceptable solution. However, in the case of  $SO$  groups with  $2N_f = N_c - 4$  or  $2N_f = N_c - 3$ , we have confinement without chiral symmetry breaking at the origin so  $w_0$  becomes an acceptable solution. From the RG equation, we observe that, for  $2N_f \geq 2N_c$ , the solution (4.10) goes to zero also so the two values for  $w_0$  coincide. For  $2N_f = 2N_c - 2, 2N_c - 3, 2N_c - 4$ , both solutions for  $w_0$  exist and they give the two branches that we have discussed in section 2. For  $2N_f \leq 2N_c - 5$ , again using the RG equation, we observe that in the left hand side of (4.9) the power of  $\mu$  becomes positive so it will become infinite in the  $\mu \rightarrow \infty$  limit. Therefore the equation (4.9) cannot have the solution of (4.11) because in the left hand side we will have the product of zero with  $\infty$  which is undetermined. So, for  $2N_f \leq 2N_c - 5$ , there is only one solution given by (4.10) which becomes infinite in this range so, as expected, there is no vacuum.

## 4.2 $SO(2N_c)$ with massless matter

After rescaling  $t$  by a factor  $\mu^{2N_c-2}$ , when we consider the case of  $2N_f$  massless quarks, the rotated curves are described by :

$$\mu^{-(2N_c-2)} v^2 \tilde{t}^2 - B(v^2, u_k) \tilde{t} + \Lambda_{N=1}^{3(2N_c-2)-2N_f} v^{2+2N_f} = 0 \quad (4.12)$$

$$\begin{aligned}\tilde{t} &= w^{2(N_c-1-N_f)}(w^2 - w_0^2)^{N_f} \\ vw &= \mu^{-1}(w^2 - w_0^2)\end{aligned}$$

where  $\tilde{t} = \mu^{2N_c-2}t$ . Since the order parameters  $u_k$  vanish in the  $\mu \rightarrow \infty$  limit, the first equation has the smooth limit given by

$$\tilde{t} = \Lambda_{N=1}^{3(2N_c-2)-2N_f} v^{2N_f-(2N_c-2)}. \quad (4.13)$$

By the RG equation

$$\Lambda_{N=1}^{3(2N_c-2)-2N_f} = \mu^{2N_c-2} \Lambda_{N=2}^{2(2N_c-2)-2N_f}. \quad (4.14)$$

we may classify the rotated branes into several cases:

- $2N_f \geq 2N_c$  The  $\mu\Lambda_{N=2}$  goes to zero for  $\mu \rightarrow \infty$  limit. Thus there is no difference between the two possible values for  $w_0$  we have discussed before.

- $2N_f \leq 2N_c - 6$  The  $\mu\Lambda_{N=2}$  goes to infinity so there is no supersymmetric vacua.

- $2N_f = 2N_c - 4$  For the value  $w_0 = \mu\Lambda_{N=2}$  we again do not have any vacuum, this case being just a continuation of the previous case. On the other hand, for  $w_0 = 0$  there exists a vacuum at the origin, and the curve is given by:

$$v = 0 \quad \tilde{t} = w^{2N_c-2} \quad (4.15)$$

- $2N_f = 2N_c - 2$  We have two possible curves, one for  $w_0 = 0$ , the other for  $w_0 = (-1)^{-N_f/2} \mu\Lambda_{N=2}$ . We obtain as in [24] the left component  $C_L$  and the right component  $C_R$ :

$$C_L \begin{cases} v = 0 \\ \tilde{t} = (w^2 - w_0^2)^{N_c-1} \end{cases} \quad C_R \begin{cases} w = 0 \\ \tilde{t} = \Lambda_{N=1}^{4N_c-4} \end{cases} \quad (4.16)$$

Of course,  $C_L$  differs as a function of the value of  $w_0$ . Furthermore, two components  $C_L$  and  $C_R$  intersect at the point  $v = w = 0$ ,  $\tilde{t} = \Lambda_{N=1}^{4N_c-4}$  for  $w_0 = (-1)^{-N_f/2} \mu\Lambda_{N=2}$  whereas  $C_L$  does not meet  $C_R$  for  $w_0 = 0$ .

### 4.3 $SO(2N_c + 1)$ with massless matter

By similar reasoning, the rotated brane for the case of  $2N_f$  massless quarks is described by:

$$\begin{aligned}\tilde{t}^2 &= w^{4N_c-2-4N_f}(w^2 - w_0^2)^{2N_f} \\ vw &= \mu^{-1}(w^2 - w_0^2)\end{aligned} \quad (4.17)$$

Again we have several cases, arising from the RG equation

$$\Lambda_{N=1}^{3(2N_c-1)-2N_f} = \mu^{2N_c-1} \Lambda_{N=2}^{2(2N_c-1)-2N_f} : \quad (4.18)$$

- $2N_f \geq 2N_c + 1$  The  $\mu\Lambda_{N=2}$  vanishes again for  $\mu \rightarrow \infty$  limit. Thus  $w_0 = \mu\Lambda_{N=2}$  goes to zero and the two branches unify for this values of  $N_f$ .

- $2N_f \leq 2N_c - 5$  The  $\mu\Lambda_{N=2}$  goes to infinity so there is no supersymmetric vacuum.

- $2N_f = 2N_c - 3$  For  $w_0 = \mu\Lambda_{N=2}$ , this again goes to infinity, and there is no vacuum. For  $w_0 = 0$  there is a vacuum at the origin, where there are additional massless field, besides the meson. The curve describing the vacuum is

$$v = 0 \quad \tilde{t}^2 = w^{4N_c-2} \quad (4.19)$$

- $2N_f = 2N_c - 1$  the  $\mu\Lambda_{N=2}$  does not depend on  $\mu$  so both solutions for  $w_0$  are possible. Again  $C_L$  depends on the value of  $w_0$ . Even if this case resembles the case of  $SU(N_c)$  for  $N_f = N_c$  and  $SO(2N_c)$  for  $2N_f = 2N_c - 2$  in the sense that  $\mu\Lambda_{N=2}$  does not depend on  $\mu$ , here the difference is that the dual is a non Abelian theory so we do not have effects like the appearance of a supplementary superpotential (as in  $SU(N_c)$  case) or the appearance of monopoles and dyons at vacua (as in  $SO(2N_c)$  case).

This completes the discussion for the rotated brane in the presence of massless matter. For discussions on dualities for theories with  $SO$  groups from M theory we refer to [33].

## 5 Conclusion

In the present work, we generalized the work of [14, 24, 22, 26] to the case of  $SO(2N_c)$  and  $SO(2N_c + 1)$  gauge groups. We showed that the brane configuration gave us information about the  $N = 1$  field theory vacua. We discussed the rotation of the M fivebrane from  $N = 2$  theory to  $N = 1$  theory and we obtained the form of the rotated curves. We want to emphasize again that we were not able to obtain the spectrum of massless particles at different vacua either in field theory or in the brane configuration picture. The same problem appeared in [29] and seems to be a common flaw of the  $N = 1$  theories obtained from  $N = 2$  by turning mass for the adjoint (in field theories) or by rotating the branes (in the brane configuration picture). A possible way to obtain information about the massless particles is to use the idea developed very recently by Strassler in [34]. In the

context of field theory, he found that the electric sources in the spinor representations can be introduced as magnetic sources in the dual nonabelian gauge theory. By turning masses for some quarks one can thus obtain information about the spectrum of particles at vacua for theories with smaller number of quarks like the theories with  $N_f = N_c - 2$  or  $N_f = N_c - 4$ . A first step towards this direction would be to generalize the construction of [21] for the  $SO$  gauge group. We hope that this problem will be solved in the near future.

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